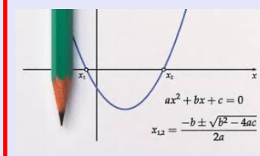
Math 125

Spring 2022

Lecture 28



Class QZ 21

Find a $\boxed{\text{quadratic equation}}$ in $\boxed{ax^2+bx+c=0}$ form with solutions $\boxed{3\pm2\sqrt{5}.}$

$x=3+2\sqrt{5}$ $\qquad$ $x=3-2\sqrt{5}$

$x-3-2\sqrt{5}=0$ $\qquad$ $x-3+2\sqrt{5}=0$

$$\underbrace{(x-3-2\sqrt{5})(x-3+2\sqrt{5})}_{\text{conjugates}}=0$$

conjugates $\longrightarrow (A-B)(A+B)=A^2-B^2$

$$(x-3)^2-(2\sqrt{5})^2=0$$

$$\boxed{(x-3)(x-3)}-4\cdot5=0$$

$$\boxed{x^2-3x-3x+9}-20=0$$

$$\boxed{x^2-6x-11=0} \checkmark\checkmark\checkmark$$

Given $(x-3)^2+(y+3)^2=9$

1) Center $(h,k)=(3,-3)$

2) Radius $r^2=9$ $\boxed{r=3}$

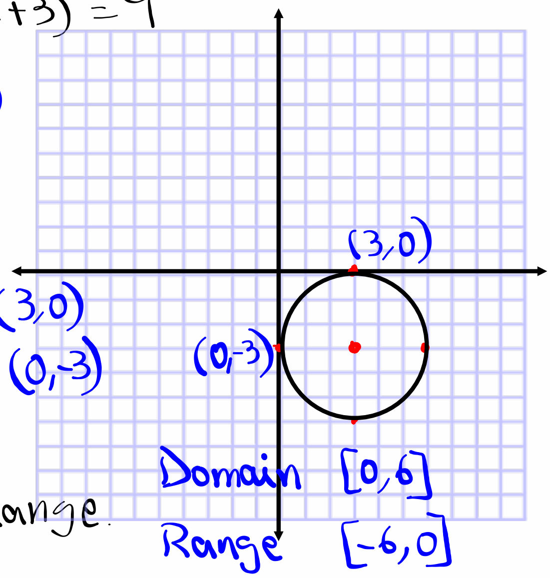
3) Draw

4) All intercepts

X-Int $(3,0)$

Y-Int $(0,-3)$

5) Discuss domain & Range



Given $\dfrac{x^2}{16}+\dfrac{(y-3)^2}{9}=1$

1) Center $(h,k)=(0,3)$

2) $a^2=16$ $a=4$

$b^2=9$ $b=3$

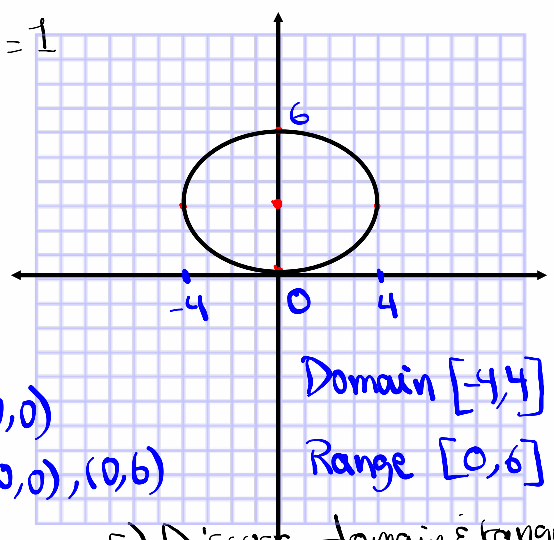
3) Draw

4) All intercepts

X-Int $(0,0)$

Y-Int $(0,0),(0,6)$

5) Discuss domain & range



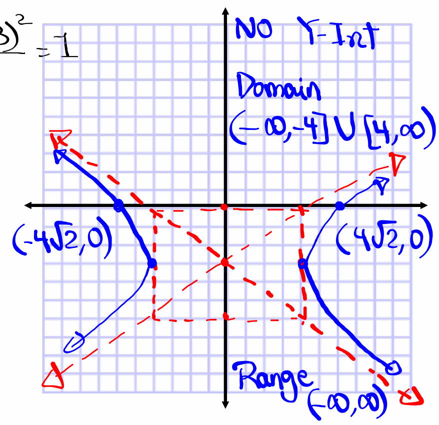
Consider $\frac{x^2}{16} - \frac{(y+3)^2}{9} = 1$

1) Center $(h,k) = (0,-3)$

2) $a^2 = 16$ $a = 4$

$b^2 = 9$ $b = 3$

3) Do a complete drawing



4) Discuss intercepts

5) Discuss domain & range

For x-Ints → Let $y=0$

$\frac{x^2}{16} - \frac{(0+3)^2}{9} = 1$

$\frac{x^2}{16} - \frac{9}{9} = 1$ → $\frac{x^2}{16} - 1 = 1$

$\frac{x^2}{16} = 2$

$x^2 = 32$ → $x = \pm\sqrt{32}$

$x = \pm\sqrt{16}\sqrt{2}$

$\boxed{x = \pm 4\sqrt{2}}$

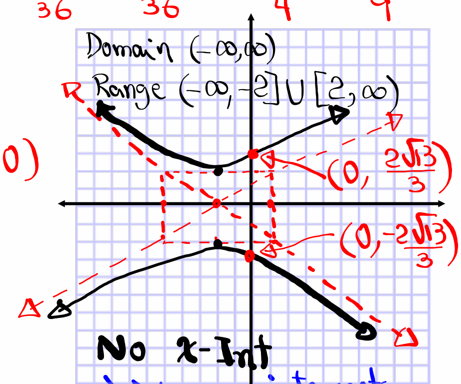
Consider $\frac{9y^2}{36} - \frac{4(x+2)^2}{36} = \frac{36}{36}$ ⇒ $\frac{y^2}{4} - \frac{(x+2)^2}{9} = 1$

1) Make RHS=1

2) Center $(h,k) = (-2,0)$

3) $a^2 = 9$ $a = 3$

$b^2 = 4$ $b = 2$



4) Draw a complete graph

5) Discuss intercepts

6) Discuss domain & Range

Y-Int → $x=0$

$\frac{y^2}{4} - \frac{(0+2)^2}{9} = 1$

$\frac{y^2}{4} - \frac{4}{9} = 1$ → LCD=36

$\frac{\cancel{36}^{9} y^2}{\cancel{4}} - \frac{\cancel{36}^{4} \cdot 4}{\cancel{9}} = 36 \cdot 1$

$9y^2 - 16 = 36$

$9y^2 = 52$ → $y^2 = \frac{52}{9}$

$y = \pm\frac{\sqrt{52}}{\sqrt{9}}$

$y = \pm\frac{2\sqrt{13}}{3}$

Consider $f(x) = 2(x-1)^2 - 2$

$f(x) = a(x-h)^2 + k$

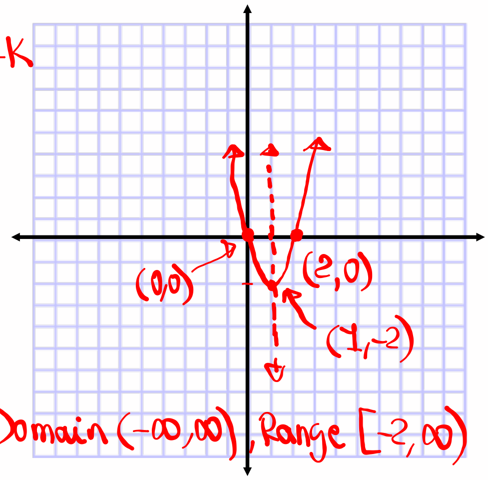
1) $a = 2$, opens up

2) Vertex $(h,k) = (1,-2)$

3) A.O.S. $x = h$ $x = 1$

4) Y-Int $(0,0)$

5) Draw



Domain $(-\infty,\infty)$, Range $[-2,\infty)$

6) Discuss domain & range

7) X-Int $(0,0), (2,0)$

Consider $f(x) = -x^2 + 4x - 4$

$f(x) = ax^2 + bx + c$

$h = \frac{-b}{2a} = \frac{-4}{2(-1)} = \frac{-4}{-2} = 2$

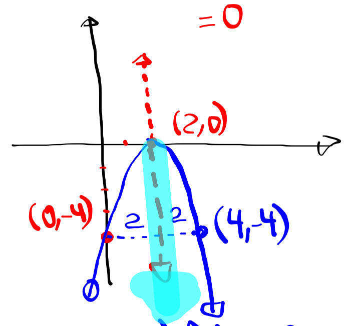
$k = -(2)^2 + 4(2) - 4$
$= -4 + 8 - 4$
$= 0$

1) a, b, and c. $a = -1$ $b = 4$ $c = -4$

↳ opens down

2) Vertex $(h,k) = (2,0)$

$h = \frac{-b}{2a}$, k = plug in h.



3) A.O.S. $x = h$ $x = 2$

4) Y-Int $(0,-4)$

5) Draw

6) Discuss domain & range.

Domain $(-\infty,\infty)$

Range $(-\infty,0]$

7) X-Int $(2,0)$

Sideway Parabolas:

$x = a(y-k)^2 + h, \; a \neq 0$

$a > 0$, $a < 0$

Vertex (h, k)

A.O.S. $y = k$

x-Int $(\;\; , 0)$, Y-Int $(0, \;\;)$

No Y-Int

Domain $[3, \infty)$

Range: $(-\infty, \infty)$

$x = (y-2)^2 + 3$

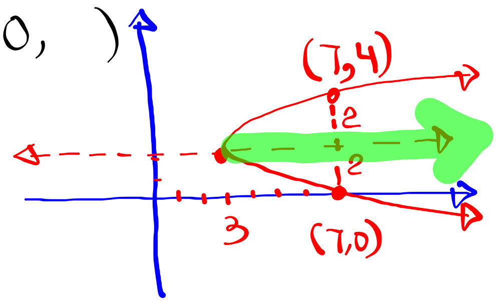
$a = 1$ opens right

$h = 3$ $\quad k = 2$

Vertex $(3, 2)$

A.O.S. $y = 2$

x-Int $(7, 0)$



Consider $x = -(y+2)^2$

$x = a(y-k)^2 + h$

1) $a = -1$ opens left

2) Vertex $(h, k) = (0, -2)$

3) A.O.S. $y = k$ $\quad y = -2$

4) x-Int $(-4, 0)$

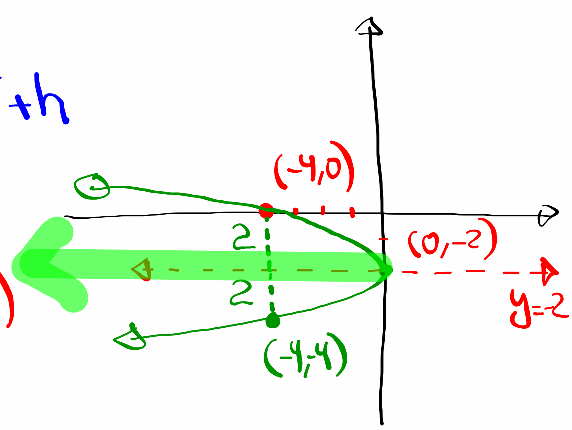
5) Draw

6) Domain & Range

7) Y-Int $(0, -2)$

Domain: $(-\infty, 0]$

Range: $(-\infty, \infty)$



Sideway Parabolas:

$x = ay^2 + by + c \ , a \neq 0$

$a > 0$ ↻ , $a < 0$ ↺

Vertex (h, k)

$k = \frac{-b}{2a}$

$h =$ Plug in k

A.O.S. $y = k$

x-Int $(\ , 0)$

Y-Int $(0, \ )$

Y-Int $(0,0), (0,4)$

Domain: $[-4, \infty)$

Range: $(-\infty, \infty)$

$x = y^2 - 4y$

$a = 1$ opens right

$b = -4$

$c = 0$

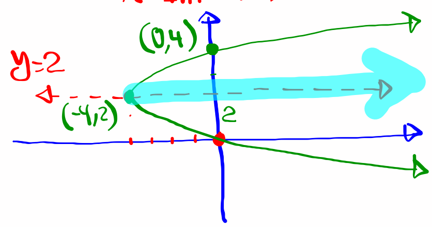
$k = \frac{-b}{2a} = \frac{-(-4)}{2(1)} = \frac{4}{2} = 2$

$h = 2^2 - 4(2) = 4 - 8 = -4$

Vertex $(-4, 2)$

A.O.S. $y = 2$

x-Int $(0, 0)$



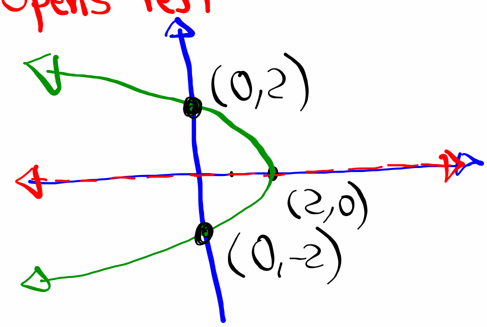
Consider $x = \frac{-1}{2} y^2 + 2$

$x = ay^2 + by + c$

$a = \frac{-1}{2}$ opens left

$b = 0$

$c = 2$



$k = \frac{-b}{2a} = \frac{-0}{2(\frac{-1}{2})} = \frac{-0}{-1} = 0$

$h = \frac{-1}{2}(0)^2 + 2 = 2$

Vertex $(2, 0)$

A.O.S. $y = 0$

x-Int $(2, 0)$

Range $(-\infty, \infty)$

Domain $(-\infty, 2]$

Y-Int $(0, \ )$

$x = 0 \qquad \frac{-1}{2} y^2 + 2 = 0$

$-y^2 + 4 = 0$

$-y^2 = -4$

$y^2 = 4$

$y = \pm\sqrt{4}$

$y = \pm 2$

Consider $x = y^2 - 6y + 10$

1) $a = 1$, $b = -6$, $c = 10$

↳ opens Right

2) Vertex $(h, k) = (1, 3)$

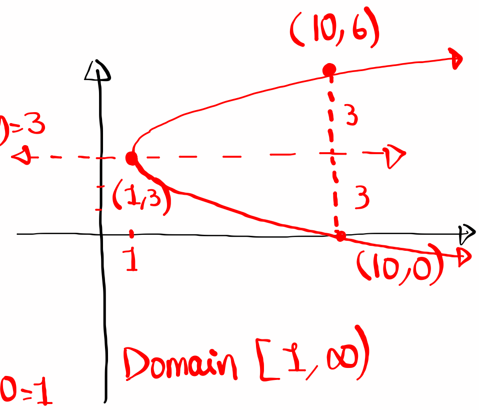
$k = \frac{-b}{2a} = \frac{-(-6)}{2(1)} = \frac{6}{2} = 3$

$h =$ Plug in K $h = 3^2 - 6(3) + 10 = 1$

3) A.O.S. $y = k$ $y = 3$

4) x-Int $(10, 0)$

5) Y-Int



Domain $[1, \infty)$

Range: $(-\infty, \infty)$

Y-Int: None

5) Draw

6) Domain & Range

SG → 30

! Factorial

$n! = n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$

$0! = 1$

$4! = 4 \cdot 3 \cdot 2 \cdot 1 = \boxed{24}$

$6! - 3! = \underbrace{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}_{} - \underbrace{3 \cdot 2 \cdot 1}_{}$

$= 720 - 6 = \boxed{714}$

$\frac{8!}{4!} = \frac{\overset{56}{8 \cdot 7} \cdot \overset{30}{6 \cdot 5} \cdot \cancel{4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{4 \cdot 3 \cdot 2 \cdot 1}} = 56 \cdot 30 = \boxed{1680}$

$\frac{9!}{6! \cdot 3!} = \frac{\overset{3}{\cancel{9}} \cdot \overset{4}{\cancel{8}} \cdot 7 \cdot \cancel{6!}}{\cancel{6!} \cdot \cancel{3} \cdot \cancel{2} \cdot 1} = 3 \cdot 4 \cdot 7 = \boxed{84}$

Combination Formula

$${}_nC_r = \frac{n!}{r!\cdot(n-r)!}$$

$${}_5C_2 = \frac{5!}{2!\cdot(5-2)!} = \frac{5!}{2!\cdot 3!} = \frac{5\cdot \overset{2}{\cancel{4}}\cdot\cancel{3!}}{\cancel{2}\cdot 1\cdot\cancel{3!}} = \boxed{10}$$

$${}_7C_3 = \frac{7!}{3!\cdot(7-3)!} = \frac{7!}{3!\cdot 4!} = \frac{7\cdot\cancel{6}\cdot 5\cdot\cancel{4!}}{\cancel{3}\cdot\cancel{2}\cdot 1\cdot\cancel{4!}} = \boxed{35}$$

$${}_8C_5 = \frac{8!}{5!\cdot(8-5)!} = \frac{8!}{5!\cdot 3!} = \frac{8\cdot 7\cdot\cancel{6}\cdot\cancel{5!}}{\cancel{5!}\cdot\cancel{3}\cdot\cancel{2}\cdot 1} = \boxed{56}$$

Permutation Formula

$${}_nP_r = \frac{n!}{(n-r)!}$$

$${}_6P_2 = \frac{6!}{(6-2)!} = \frac{6!}{4!} = \frac{6\cdot 5\cdot\cancel{4!}}{\cancel{4!}} = \boxed{30}$$

$${}_7P_3 = \frac{7!}{(7-3)!} = \frac{7!}{4!} = \frac{7\cdot 6\cdot 5\cdot\cancel{4!}}{\cancel{4!}} = \boxed{210}$$

Binomial Coef.:

$$\binom{n}{r} = {}_{n}C_{r} = \frac{n!}{r!\cdot(n-r)!}$$

$$\binom{9}{4} = {}_{9}C_{4} = \frac{9!}{4!\cdot 5!} = \frac{9\cdot \overset{2}{\cancel{8}}\cdot 7\cdot \cancel{6}\cdot \cancel{5!}}{\cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot 1\cdot \cancel{5!}}$$

$$= 9\cdot 2\cdot 7 = \boxed{126}$$

$$\binom{12}{5} = {}_{12}C_{5} = \frac{12!}{5!\cdot 7!} = \frac{\cancel{12}\cdot 11\cdot \cancel{10}\cdot 9\cdot 8\cdot \cancel{7!}}{\cancel{5}\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot 1\cdot \cancel{7!}}$$

$$= 11\cdot 9\cdot 8 = \boxed{792}$$

Pascal's Triangle

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

Binomial Expansion $\qquad a+b \neq 0$

$(a+b)^0 = 1$

$(a+b)^1 = a+b$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

$(a+b)^n$

1) There will be $n+1$ terms

2) Degree of each term is n.

Sum of exponents

3) First term: a^n $\qquad$ last term: b^n

Powers of a reduce, Powers of b increase

$(a+b)^n =$

$\binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \cdots + b^n$

Expand $(a+b)^5$

$$\binom{5}{0}a^5+\binom{5}{1}a^4b+\binom{5}{2}a^3b^2+\binom{5}{3}a^2b^3+\binom{5}{4}ab^4+\binom{5}{5}b^5$$

1 5 10 10 5 1

Final exam: Thursday June 2, 2022

7:00 - 9:00